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Differential Calculus (Curvature)

Q.1 Find the radius of curvature of the curve $y^2 = x^2 \cdot \frac{a+x}{a-x}$ at the point $(-a, 0)$.

Soln: Let us shift the origin to the given point $(-a,0)$.

For this, putting $x = x-a$ and $y = y+0$ in the given eqn

$$y^2 = x^2 \cdot \frac{a+x}{a-x} \text{ of the curve.}$$

$$\therefore \quad y^2 = (x-a)^2 \frac{a+x-a}{a-(x-a)} = \frac{x(x-a)^2}{2a-x}$$

$$\Rightarrow \quad y^2(2a-x) = x(x-a)^2 \text{ ——— (I)}$$

We have to get the tangent at the origin, equate the lowest degree terms of I to zero,

$$\therefore \quad xa^2 = 0 \Rightarrow x = 0$$

This means that y-axis is tangent to the curve (I) at the origin.

$$\therefore \quad \rho = \lim_{\substack{x \to 0 \\ y \to 0}} \frac{y^2}{2x}$$

From (I), $\quad \frac{y^2}{2x} = \frac{1}{2} \cdot \frac{(x-a)^2}{2a-x}$

$$\therefore \quad \lim_{\substack{x \to 0 \\ y \to 0}} \frac{y^2}{2x} = \frac{1}{2} \lim_{x \to 0} \frac{(x-a)^2}{2a-x}$$

$$\Rightarrow \quad \rho = \frac{1}{2} \cdot \frac{(-a)^2}{2a} = \frac{1}{2} \times \frac{a^2}{2a} = \frac{a}{4}$$

Hence $\rho = \frac{1}{4}a$.

Q.2: 1. Find the radius of curvature at the origin on the curve $(y-x)(y-2x) = x^2 + y^2$

Soln: First of all we have to put $y = px + q\frac{x^2}{12} + \cdots$

Where $\left(\frac{dy}{dx}\right)_{(0,0)} = p$ and $\left(\frac{d^2y}{dx^2}\right)_{(0,0)} = q$

B.Sc-part-II(H), paper-III

Date- 02-05-2020

Rest part of Q.2.

Substituting the value of y in the equation of the curve, we get

$$\left[\left(px+q\frac{x^2}{\lfloor 2}+\cdots\right)-x\right]\left[\left(px+q\frac{x^2}{\lfloor 2}+\cdots\right)-2x\right]$$

$$=x^2+\left[px+q\frac{x^2}{\lfloor 2}+\cdots\right]^3$$

Comparing the coefficients of x^2 and x^3, we get

$(p-1)(p-2)=0$ and $\frac{1}{2}[(p-1)q+q(p-2)]=1+p^3$

$\therefore$ Either $p=1$ or $p=2$

when $p=1$, we get $\frac{1}{2}q(-1)=1+1=2 \Rightarrow q=-4$

When $p=2$, we get $q=18$

Thus, when $p=1, q=-4$ $\therefore \rho=\pm\frac{(1+p^2)^{\frac{3}{2}}}{q}=\pm\frac{2\sqrt{2}}{4}=\pm\frac{\sqrt{2}}{2}=\pm\frac{1}{\sqrt{2}}$

And when $p=2, q=18$ $\therefore \rho=\pm\frac{5\sqrt{5}}{18}$

Q.3. Show that the radius of curvature of the curve $r=a\sin n\theta$ at the origin is $\frac{na}{2}$.

First method:-

Soln: At the origin, $\rho=\underset{\substack{\theta\to 0\\ r\to 0}}{Lt}\left(\frac{r}{2\theta}\right)=\underset{\theta\to 0}{Lt}\left(\frac{a\sin n\theta}{2\theta}\right)$ $[\because r=a\sin n\theta]$

$$=\underset{\theta\to 0}{Lt}\ \frac{a\sin n\theta}{n\theta}\cdot\frac{n}{2}$$

$$=\underset{\theta\to 0}{Lt}\left(\frac{\sin n\theta}{n\theta}\right)\cdot a\cdot\frac{n}{2}=\frac{na}{2}$$ Proved

Second method:

$\because r=a\sin n\theta$

$\therefore r_1=an\cos n\theta$ and $r_2=-an^2\sin n\theta$

At the origin $r=0$. $\therefore$ From the eqn of the curve $\theta=0$

$\therefore r_1=an$ and $r_2=0$

$\therefore \rho$ at the origin $=\frac{(r^2+r_1^2)^{\frac{3}{2}}}{r^2+2r_1^2-rr_2}=\frac{(an)^{3}}{2a^2n^2}=\frac{an}{2}$ Proved

Solved